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LINEAR TRANSIENT DYNAMICS OF PERTURBATIONS IN NONGEOSTROPHIC FLOWS WITH A CONSTANT VERTICAL SHEAR

1. Introduction

Stationary zonal flows with vertical shear and stratification are frequently found in the atmosphere and ocean. It is now well established that perturbations in these flows (e.g. coherent structures) gain the mean flow energy due to the linear mechanism. In other words, they emerge from the analysis of governing dynamical equations linearized about the mean shear flow ([1],[19],[5],[11],[12],[15]). However, the operators occurring in such linearized dynamical equations are nonnormal, due to the shear of the mean velocity field, and corresponding eigenfunctions are nonorthogonal and strongly interfere ([38]), thereby very much complicating the comprehension of perturbation dynamics. For this reason classical canonical spectral (modal) method of hydrodynamics, i.e., spectral expansion of perturbed quantities in time, usually does not (actually it is unable to) predict and characterize finite-time/transient phenomena (existing even in the simplest shear flows).

Among finite-time phenomenon that occur due to the nonnormality of governing operators one can distinguish strong linear transient growth of perturbations in smooth (without inflection point) shear flows ([39],[11],[16],[22],[40],[5]) including the case of stratified shear flows ([13],[1]) and linear coupling of different perturbation modes. The mode coupling, yet not fully appreciated by the meteorological community has originally been revealed and thoroughly described in the simplest nonrotating shear flow in [6] and in zonal nongeostrophic flows with horizontal shear in [27]. In these papers the linear conversion of vortical perturbations into wave ones is investigated. Traditionally, coupling among different perturbation modes is associated with nonlinear effects, whereas as shown here and above two papers it persists even in the linear theory provided there is a shear of mean velocity field. These two basic linear phenomena originating from the velocity shear, or nonnormality should be taken into consideration for the proper determination of turbulent spectra and the structure of coherent motions.

In the present paper we investigate the linear dynamics of perturbations in unbounded zonal inviscid flows with a constant vertical shear of velocity, when a fluid is incompressible and density is stably stratified along the vertical and meridional (or spanwise) directions. Such a flow can be considered as a model of boundary layer or of a thermal wind. We will show below that in such a flow the meridional stratification of the density arises naturally in the presence of Coriolis force and vertical shear. One can look on this in another way, in a rotating fluid the vertical shear of zonal mean velocity is maintained by the meridional gradient of density (temperature). In the case of the latter interpretation we arrive at the definition of a thermal wind.

Perturbations to this type of flow can be divided into two main classes: symmetric that are independent of the streamwise coordinate and nonsymmetric with streamwise dependence. The theory of symmetric perturbations is quite well developed by now. Necessary and sufficient conditions for symmetric stability are formulated in fundamental papers [44],[45],[36],[18]. Linear theory of the symmetric instability is presented in papers ([3],[41],[9],[43]). In these papers it is shown, in particular, that flows with a constant vertical shear are symmetrically unstable if Richardson number $\mathrm{Ri} < 1$. Subsequent evolution of the symmetric instability leads to the emergence of billows in the plane perpendicular to the flow. Active development of symmetric instability theory is connected with the problems of generation of the Gadlay cells in the atmosphere, frontal cloud bands, band structure of the Jupiter's atmosphere ([29],[30],[21],[44],[45],[42]). There are a number of important results obtained in the theory of nonlinear symmetric instability ([25],[26],[8],[32],[33],[28]). However, symmetric perturbations do not display the above mentioned transient phenomena. All these phenomena arise only in the case of nonsymmetric perturbations. So, in this paper we restrict our attention to nonsymmetric perturbations only, however in the case where the condition for symmetric instability, $\mathrm{Ri} < 1$, is met. We show that these perturbations can be classified into two fundamental types/modes – wave and vortex – according to the value of potential vorticity (PV). Wave perturbations are oscillatory and have zero PV, whereas vortex perturbations are aperiodic and have nonzero PV. Such a classification is analogous to that accepted in the adjustment theory ([35], [4], [20], [46],[2]). In the considered case, these two modes appear to be linearly coupled for a certain range of characteristic parameters of the problem. Specifically, initially imposed vortex mode perturbations evolving in the flow gain the mean flow energy and generate wave mode perturbations. This linear coupling is due to the Coriolis force and shear, or nonnormality. Here we describe in detail this mode coupling phenomenon. In the analysis we employ a so-called nonmodal approach we trace the linear dynamics of spatial Fourier harmonics of perturbations in time.

The paper is organized as follows: mathematical formalism is presented in Sec. II. The dynamics of non-symmetric wave and vortical perturbations are studied numerically in Sec. III. Conclusions are given in Sec. IV.

2. Mathematical formalism

The dynamics of a rotating stratified incompressible fluid on an *f*-plane in the Boussinesq approximation is governed by the following set of equations:

$$\frac{d\mathbf{V}}{dt} + f[\boldsymbol{n}, \mathbf{V}] = -\nabla P + \sigma \boldsymbol{n}, \qquad \frac{d\sigma}{dt} = 0, \qquad \mathrm{div}\mathbf{V} = 0, \tag{1}$$

where $\mathbf{V}$ is the velocity with components u, v, w along the zonal (*x*), meridional (*y*) and vertical (*z*) axes respectively, $\sigma \equiv -g\rho'/\rho_*$ is the buoyancy, ρ' is the deviation of density from the background value $\rho_* = const$, $P \equiv p'/\rho_*$, p' is the deviation of pressure from the hydrostatic value, g is the gravitational acceleration, f is the

Coriolis parameter, $\boldsymbol{n}$ is the unit vector along the vertical z-axis, $d/dt = \partial/\partial t + (\mathbf{V}, \nabla)$ is the total derivative operator. From Equations (1) follows the conservation of potential vorticity ([20],[37]):

$$\frac{dq}{dt} = 0, \quad q \equiv (\mathrm{curl}\mathbf{V} + f\boldsymbol{n}, \nabla\sigma). \qquad (2)$$

There exists a class of exact analytic solutions of Equations (1) describing geostrophic zonal flows with velocity shear along the vertical and meridional directions:

$$\mathbf{V_0} = (\bar{u}(y,z), 0, 0), \quad P = \bar{P}(y,z), \quad \sigma = \bar{\sigma}(y,z), \quad f\bar{u} = -\partial\bar{P}/\partial y, \quad \bar{\sigma} = \partial\bar{P}/\partial z. \qquad (3)$$

From Equations (3) follows the equation of thermal wind $f\,\partial\bar{u}/\partial z = -\partial\bar{\sigma}/\partial y$. The vertical and meridional gradients of $\bar{\sigma}$, denoted respectively as N^2 (Brunt-Vaisala frequency) and S^2, are assumed to be spatially constant.

Let us investigate the stability of an unbounded geostrophic zonal flow with a vertical shear of velocity given by $A \equiv S^2/f$ (Eady flow):

$$\bar{u} = Az, \quad \bar{\sigma} = N^2 z - S^2 y. \qquad (4)$$

Assuming $\mathbf{V} = \mathbf{V}_0 + \mathbf{V}'$, $\sigma = \bar{\sigma} + \sigma'$, $P = \bar{P} + P'$ in Equations (1), in the linear approximation for the small deviations $\mathbf{V}'$, σ', P' from (4) we obtain the system (primes are omitted):

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + Az\frac{\partial}{\partial x}.$$

From Equations (5) one can easily derive the conservation law for linearized potential vorticity:

$$\frac{Dq}{Dt} = 0, \quad q = \frac{1}{f}\left(S^2\frac{\partial\sigma}{\partial y} + f^2\frac{\partial\sigma}{\partial z}\right) + N^2\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - S^2\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \qquad (6)$$

representing the linearized form of Equation (2) and playing an important role in the subsequent analysis. Following the standard method of non-modal approach ([10],[6],[1],[27]), we introduce the spatial Fourier harmonics of perturbations with a time dependent, or "drifting" in wavenumber (k, l,m) space, vertical wavenumber $m(t)$:

$$(u, v, w, \sigma, P, q) = (\tilde{u}(t), \tilde{v}(t), \tilde{w}(t), \tilde{\sigma}(t), \tilde{P}(t), \tilde{q})\exp(ikx + ily + im(t)z) \qquad (7)$$

where $k, l, m(t) \equiv m - Akt$ are the zonal, meridional and vertical wavenumbers respectively. $\tilde{u}(t), \tilde{v}(t), \tilde{w}(t), \tilde{\sigma}(t), \tilde{P}(t)$ are the amplitudes depending only on time. Note that the amplitude $\tilde{q}$ does not depend on time, because of the conservation of potential vorticity. In the physical variables (x, y, z), the representation (7) describes a harmonic plane wave with a time-dependent amplitude and phase $\theta = kx + ly + m(t)z$. As the meridional wavenumber $m(t)$ is time-dependent, the surface of constant phase rotates and becomes nearly parallel to the horizontal (x, y) plane in the limit $t \to \infty$. Substituting Equations (7) into Equations (5)-(6), making changes: $At \to t$, $(\tilde{u}/u_0, \tilde{v}/u_0, \tilde{w}/u_0) \to (\tilde{u}, \tilde{v}, \tilde{w})$, $\tilde{\sigma}/Au_0 \to \tilde{\sigma}$, $-ik\tilde{P}/Au_0 \to \tilde{P}$, $-i\tilde{q}/u_0 kA^2 \to \tilde{q}$ where u_0 is a typical mean flow velocity, and introducing notations: nondimensional Coriolis parameter $T \equiv 1/\mathrm{Ro} = f/A$, Richardson number $\mathrm{Ri} \equiv N^2/A^2$, $a(t) \equiv m(t)/k$, $b \equiv l/k$ $(k \neq 0)$, for the time-dependent amplitudes we finally get:

$$\frac{d\tilde{u}}{dt} - T\tilde{v} + \tilde{w} = \tilde{P}, \quad \frac{d\tilde{v}}{dt} + T\tilde{u} = b\tilde{P}, \quad \frac{d\tilde{w}}{dt} - \tilde{\sigma} = a(t)\tilde{P}, \quad \frac{d\tilde{\sigma}}{dt} - T\tilde{v} + \mathrm{Ri}\tilde{w} = 0, \quad \tilde{u} + b\tilde{v} + a(t)\tilde{w} = 0, \qquad (8)$$

and the conservation of potential vorticity

$$(b + Ta(t))\tilde{\sigma} + \mathrm{Ri}(\tilde{v} - b\tilde{u}) + T(\tilde{w} - a(t)\tilde{u}) = \tilde{q} = const. \qquad (9)$$

Because from here on we use only these amplitudes everywhere, tildes over them will be omitted. Note that the perturbation dynamics depends on the ratios of wavenumbers $a(t)$ and b, since there is no characteristic length scale in the problem. As mentioned in the Introduction, we focus on Richardson numbers in the range $\mathrm{Ri} < 1$, which may occur in fronts and jet streaks. If we take H = 10 km for the height of the earth atmosphere and the characteristic of large scale flows velocity $u_0 = 10ms^{-1}$, for the shear parameter we find $A = u_0/H = 10^{-3}s^{-1}$ and, consequently, for $T = f/A = 0.1, \mathrm{Ro} = 10$ $(f = 10^{-4}s^{-1})$.

Using Equation (9) and after some algebra, system (8) reduces to the following second order inhomogeneous differential equation for $h \equiv \sigma\sqrt{s^2(t)/B(t)}$:

$$\frac{d^2h}{dt^2} + w^2(a(t), b, \mathrm{Ri}, T)h = qF(t), \qquad (10)$$

where $$s^2(t) \equiv 1 + b^2 + a^2(t), \quad B(t) \equiv (\mathrm{Ri}(1+b^2) + Tba(t))^2 + T^2 s^2(t).$$

The form of w^2 and the coupling function *F(t)* as well as the equations expressing *u, v,w, P* through σ and $d\sigma/dt$ are given in the Appendix.

The combined effect of the earth rotation (Coriolis parameter), vertical shear and stratification leads to the emergence of broad conic unstable regions (where $w^2 < 0$) together with much smaller central unstable regions only for $\mathrm{Ri} < 1$. These unstable regions shrink and become vanishing with increasing Ri. As we will see below, their role in the dynamics is much reduced starting from Ri ; 1. Because of the shear the square of the frequency w^2 becomes time-dependent; drifting in (*a, b*) plane spatial Fourier harmonic of perturbation crosses alternatively stable and unstable regions which is reflected in the evolution plots of various perturbed quantities presented below.

The non-dimensional perturbation energy density of a single plane wave may be given in terms of the Fourier amplitudes by

$$E = \frac{1}{2}(|u|^2 + |v|^2 + |w|^2) + \frac{|\sigma|^2}{2\mathrm{Ri}}.$$

The first term in this expression represents the kinetic energy and the second one the potential energy of perturbations. The evolution equation for the energy is straightforward to derive:

$$\frac{dE}{dt} = -\frac{1}{2}(uw^* + wu^*) + \frac{T}{2\mathrm{Ri}}(v\sigma^* + \sigma v^*),$$

where the asterisk denotes the complex conjugate. The first term describes energy gain (or loss) by perturbations due to Reynolds stress, while the second term is energy gain (or loss) due to meridional heat flux [24]. Energy extraction from the mean flow by perturbations is possible solely due to shear; in the shearless limit, or in the absence of meridional stratification, the perturbation energy is a conserved quantity.

Equation (10) describes two different modes/types of perturbations:

(1) ageostrophic inertio-gravity wave mode $h^{(w)}$; that is oscillatory and is determined by a general solution of the corresponding homogeneous equation and has zero potential vorticity,

(2) geostrophic vortex mode $h^{(v)}$; that is aperiodic, originating from the equation inhomogeneity $qF(t)$, and is associated with a non-oscillatory part of a particular solution of the inhomogeneous equation. In the shearless limit this mode is independent of time and characterized by zero density and vertical velocity perturbations and non-zero potential vorticity, however, in the presence of shear and rotation it acquires non-zero density and vertical velocity perturbations as well. Therefore, vortex/aperiodic mode is uniquely determined. The amplitude of the vortex mode is proportional to *q* and goes to zero when $q = 0$. Such a separation of modes is appropriate only far from the unstable regions where the adiabatic (WKBJ) condition with respect to time, $\left| w'(t)/w^2(t) \right| = 1$, holds.

Performed here classification of perturbation modes into two types – waves and vortices – depending on the value of potential vorticity is analogous to that accepted in the classical linear theory of geostrophic adjustment ([35],[4],[20],[2],[46]).

In the following we will keep to the physical standpoint of separation of perturbation modes. Thus, the general solution of Equations (8) can be expressed as a superposition of oscillatory/wave and aperiodic/vortex components and we write:

$$u = u^{(\mathrm{w})} + u^{(\mathrm{v})}, \quad v = v^{(\mathrm{w})} + v^{(\mathrm{v})},$$

$$w = w^{(\mathrm{w})} + w^{(\mathrm{v})}, \quad \sigma = \sigma^{(\mathrm{w})} + \sigma^{(\mathrm{v})}.$$

In fact, the (modified) initial value problem is solved by Equation (10) (or, equivalently, by Equations (8)). The character of the subsequent dynamical evolution depends on the mode of perturbation inserted initially in Equations (8) or (10): a pure wave mode (without admixes of aperiodic vortices) or a pure aperiodic vortex mode (without admixes of waves). The primary aim of the present paper is to investigate the properties of the vortex mode, in particular, the generation of the wave mode by the vortex mode and the transient amplification of the vortex mode itself until the point of wave generation for small Richardson numbers, $\mathrm{Ri} \le 1$. For large Richardson numbers very weak coupling was found between ageostropic inertio-gravity waves and geostrophic vortical perturbations ([23]), while for intermediate Richardson numbers this coupling is quite appreciable ([24]): inertio-gravity waves first undergo transient amplification and then generate vortical perturbations. Here we show that for small Richardson numbers the transient amplification is even stronger and the conversion of geostrophic vortical perturbations into ageostrophic inertio-gravity waves can also take place. Due to the strict conservation of PV in the present problem, wave mode can not generate vortex mode, because the former is characterized by zero PV that is conserved during the entire course of evolution.

3. Numerical analysis of vortex mode evolution: generation of waves by the vortex mode

In this subsection we describe the linear mechanism of the emergence of oscillations from aperiodic perturbations initially imposed onto a flow, or physically, the generation of the wave mode by the vortex mode. In order to initially (at *t* = 0) impose pure vortex mode perturbations we use the approximate solution that can be derived from Equation (10):

$$h^{(\mathrm{v})}(t) = \frac{qF(t)}{w^2(t)}. \qquad (11)$$

Corresponding values $u^{(\mathrm{v})}(t), v^{(\mathrm{v})}(t), w^{(\mathrm{v})}(t), \sigma^{(\mathrm{v})}(t), P^{(\mathrm{v})}(t)$ can be readily found from $h^{(\mathrm{v})}(t)$ and its time derivative using expressions provided in the appendix. It is clear that these values are proportional to q and vanish in the limit $q = 0$. This solution describing the vortex mode is valid when (i) the time scale of variation of the vortical perturbations is much larger than $2\pi/w$ and (ii) the adiabatic condition $\left|w'(t)/w^2(t)\right| = 1$ holds. These two constraints are best fulfilled far outside the unstable regions. In fact, a rigorous separation of the vortex mode is possible just under these two conditions and requires asymptotic methods ([31],[34],[17]). However, we find the above approximate solution fairly satisfactory for choosing initial conditions corresponding to the pure vortex mode in our numerical calculations. Suppose we initially take $a(0)$ and b well outside the unstable regions and, in addition, $a(0)$ is such that the wavenumber $a(t)$ starting from the adiabatic region, remains there never crossing the unstable (non-adiabatic) regions. In this situation, the vortex mode dynamics is rather simple and uninteresting; it gradually dies down without generating waves and experiencing transient amplification. Thus, crossing and, therefore, the existence of a non-adiabatic region is necessary for the described here non-resonant wave generation. We consider only such values of $a(0)$ when the wavenumber $a(t)$ starting from the adiabatic region during its drift passes through the unstable regions. Because $a(t)$ drifts opposite the a-axis, such $a(0)$ should be to the right of the unstable regions.

First consider the case of an initially imposed vortex mode perturbation with such $a(0)$ that during the drift crosses the central unstable region. The subsequent evolution of the perturbed quantities u, v, w, σ, P and the normalized energy $E/E(0)$ corresponding to this situation are shown in Fig.1. In the beginning, being in the adiabatic region, the imposed vortex mode extracts energy from the mean shear flow due to the non-normality and grows algebraically, but retains its aperiodic nature. Then on approaching the unstable region the dynamics becomes non-adiabatic. From this moment the transient amplification gains more strength, because it enters the unstable region and simultaneously oscillations begin to emerge, i.e., we observe the appearance of waves – at this stage the linear coupling of the vortex and wave modes is at work. Thus, it turns out that the linear dynamics of the vortex mode perturbation is followed by the wave generation. In the the unstable region timescales of the vortex and wave modes are comparable and the modes are not separable/distinguishable, we have some mix of aperiodic and oscillatory modes. At later times, with moving away from the unstable regions when the adiabatic approximation is applicable again, the dynamics of the vortex and wave modes become decoupled and the modes get cleanly separated: time scale of the wave mode (equal to f^{-1}) becomes much shorter than that of the vortex mode. One can formally divide the energy evolution into two phases: the first phase represents the transient amplification (both due to the non-normality and to the central unstable region) of the originally imposed vortex mode perturbation and simultaneous excitation (and also further amplification) of the corresponding wave mode perturbation. The second (asymptotic) phase represents newly generated wave perturbations with the oscillating about a constant value energy. The contribution of the vortex mode energy to the total perturbation energy gradually falls off after transient amplification and wave generation and is zero at asymptotically large times. By contrast, the wave mode perturbation retains its energy and is not decreasing at large times. One can say that vortex mode perturbations act as a mediator between the mean flow and waves. The energy needed for the wave excitation is extracted from the mean flow with the help of the vortex mode.

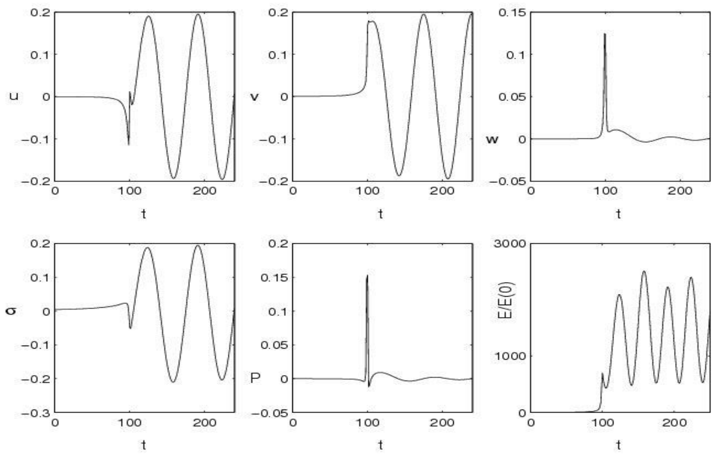

Figure 1: Evolution of the perturbed quantities u, v, w, σ, P and $E/E(0)$ with initial values corresponding to the initially imposed pure vortex mode for $a(0) = 100$, $b = 0.3$, Ri = 0.3 and Ri = 10. Before reaching the central unstable region, the vortex mode gains energy from the mean flow and exponentially amplifies retaining its aperiodic nature. In the central unstable region (near $t_* = a(0)$) the emergence of the wave mode is observed. Note also that the initially imposed vortex mode having in the beginning small density and vertical velocity perturbations develops them in the course of evolution.

Note also that the initially imposed vortex mode, because of its geostrophic nature, having in the beginning almost zero density and vertical velocity perturbations develops them in the course of evolution. The nature of the wave generation phenomenon (also called conversion of vortices into waves) is described in detail for the simplest compressible shear flow in [6] and in [7] and in the meteorological context in [27]. As mentioned, another type of mode coupling – generation of vortex mode by the wave mode is described in [24]. Thus, the linear mode coupling is characteristic of zonal nongeostrophic shear flows.

The case of b = 5 is shown in Fig. 2. The evolution picture is qualitatively similar to that in the above case, except that the wave generation takes considerably more time because of the broader unstable regions and, as it is evident from Fig. 2, may be considerably larger than growth in the case presented on Fig. 1. Thus, in both cases the linear generation of wave mode perturbations by vortex mode ones is associated with the violation of the adiabatic condition. Wave generation is gradually reduced with increasing Ri. Fig. 3 provides support for this conclusion: transient amplification of vortex mode energy for fixed b and a(0) increases orders of magnitude with decreasing Ri. For Ri = 3, 10 (and also for larger values) wave generation is absent; energy curve has no plateau indicative of generated waves for asymptotically large imes, instead it falls off at later time.

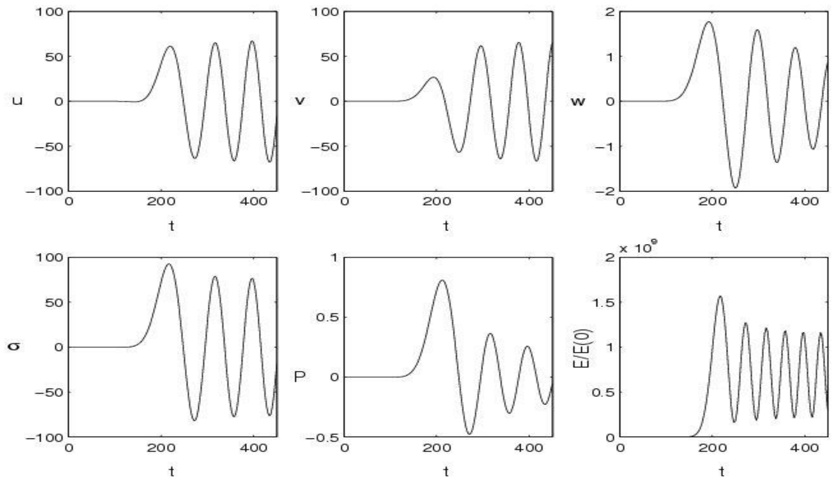

Figure 2: Same as in Fig.1, but at b = 5. In this case the wave generation occurs in the broad unstable regions. This also seen in that near $t_* = a(0)$ there is no sign of wave emergence; instead it starts later (at about t_1 = 115) on approaching the broad unstable region and continues longer time until it leaves this region (at about t_2 = 200). t_1 and t_2 are the turning points of w^2 for a fixed b, Ri. Consequently, the exponential transient growth of the vortex mode in these unstable regions is of much longer duration.

4. Conclusions

In the present paper we have investigated the linear dynamics of non-symmetric vortex mode perturbations imposed on zonal nongeostrophic flows with a constant vertical shear using the non-modal approach and numerical analysis. It has been shown that the important feature of non-symmetric perturbations is the conservation of potential vorticity. Depending on the value of the potential vorticity, the perturbations are classified as wave/oscillatory perturbations (with q = 0) and vortex/aperiodic perturbations (with $q \neq 0$).

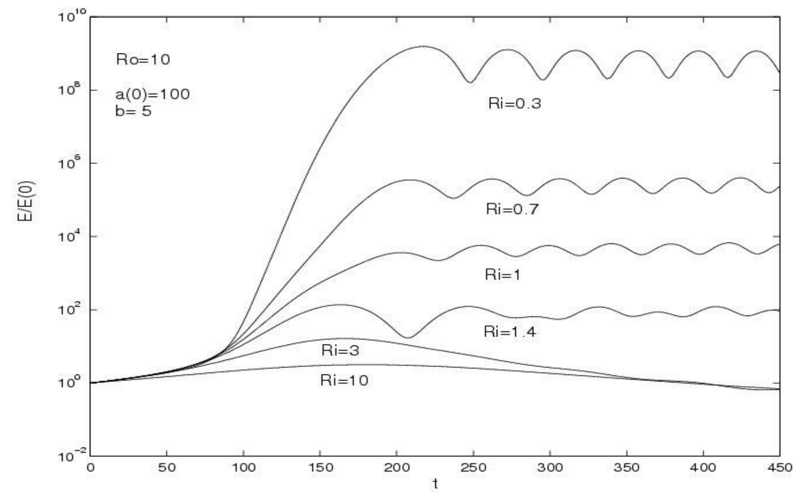

Figure 3: Evolution of perturbation energy for various Ri = 0.3, 0.7, 1, 1.4, 3, 10 at fixed b = 5, a(0) = 100 and for initially imposed vortex ode. Transient growth increases with decreasing Ri. Plateau in the energy curves for Ri = 0.3, 0.7, 1, 1.4 corresponds to generated waves, while decreasing at later times energy curves for Ri = 3, 10 correspond to asymptotically vanishing vortex mode that no longer generates waves.

It has been demonstrated that Coriolis parameter, or the earth rotation greatly changes the dynamical picture: it leads to new broad unstable regions in contrast to the non-rotating case where only relatively small central unstable region exists for Ri < 1. Due to the unstable regions, the violation of adiabatic condition is possible that, in turn, leads to linear mode coupling phenomenon, in particular, the generation of wave motions by slowly varying vortical perturbations. It has also been shown that because of the earth rotation the energy of wave perturbations does not decay at asymptotically large times as opposed to that in the non-rotating case. However, vortex mode perturbations decay gradually after wave generation (note that they remain unchanged with time in the non-rotating case). They first extract energy from the mean flow, put this energy into wave generation and then die down. An analogous wave generation occurs in flows with horizontal velocity shear except that in that case the energy of generated waves increases linearly at large times [27].

5. Appendix

The time-dependent coefficients $w^2(a(t),b,\mathrm{Ri},T)$ and F(t) of Equation (10) are given by

$$w^2(a(t),b,\mathrm{Ri},T)=\frac{a^2(t)}{s^4(t)}+\frac{2b^2}{s^2(t)}+\frac{4Tba(t)}{s^2(t)}+\frac{T^2a^2(t)}{s^2(t)}-\frac{1}{s^2(t)}+\frac{\mathrm{Ri}(1+b^2)}{s^2(t)}+\frac{T^2(1+b^2)}{B(t)}-$$

$$-\frac{3T^2(1+b^2)^2(b\mathrm{Ri}+Ta(t))^2}{B^2(t)}+\frac{2(1+b^2)(b\mathrm{Ri}+Ta(t))}{s^2(t)B(t)}\times\left[Ta(t)-(b+Ta(t))(\mathrm{Ri}(1+b^2)+Tba(t))\right],$$

$$F(t)=\sqrt{\frac{s^2(t)}{B(t)}}\left(\frac{2b+Ta(t)}{s^2(t)}-\frac{2(1+b^2)(b\mathrm{Ri}+Ta(t))(\mathrm{Ri}(1+b^2)+Tba(t))}{s^2(t)B(t)}\right)$$

The expressions for u,v,w,P in terms of σ and $d\sigma/dt$ are:

$$u=\frac{1}{G}\left[(b\mathrm{Ri}+Ta(t))(-q+(b+Ta(t))\sigma)+(a(t)\mathrm{Ri}-Tb)\frac{d\sigma}{dt}\right],$$

$$v=\frac{1}{G}\left[q\mathrm{Ri}-\mathrm{Ri}(b+Ta(t))\sigma+(T(1+a^2(t))+\mathrm{Ri}ba(t))\frac{d\sigma}{dt}\right],$$

$$w=\frac{1}{G}\left[Tq-T(b+Ta(t))\sigma-(\mathrm{Ri}(1+b^2)+Tba(t))\frac{d\sigma}{dt}\right],$$

$$P=-\frac{a(t)\sigma+T(1+b^2)v+(Tba(t)-2)w}{1+b^2+a^2(t)},$$

where $G=T^2+\mathrm{Ri}^2+(b\mathrm{Ri}+Ta(t))^2$.

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შეშფოთებათა წრფივი ტრანზიენტული დინამიკა მუდმივი ვერტიკალური წანაცვლების არაგეოსტროპულ ნაკადებში./ლომინაძე ჯ., ჩაგელიშვილი გ., ავსარქისოვი ვ./. ჰმი-ს შრომათა კრებული -2008.-ტ.115.-გვ. 330-343.-ინგლ.; რეზ. ქართ., ინგლ., რუს.

შესწავლილია შეშფოთებათა წრფივი დინამიკა შემოუსაზღვრელ არაგეოსტროპულ ზონალურ ნულოვანი სიბლანტის ნაკადებში სიჩქარის მუდმივი ვერტიკალური წანაცვლებით როდესაც გარემო უკუმშვადია და სიმკვრივე სტრატიფიცირებულია ვერტიკალური და მერიდიანული მიმართულებებით. ამ დინამიკის განსაკუთრებული თვისებები მჭიდრო კავშირშია წრფივი შეშფოთებათა ევოლუციის აღმწერი ოპერატორების არაორთოგონალურობასთან წანაცვლებით ნაკადებში. გაანალიზებულია კორიოლისის პარამეტრისა f (დედამიწის ბრუნვა) და ნაკადის წანაცვლების A როლი შეშფოთებათა დინამიკაში (არამდგრადობაში). ეს ორი ფაქტორი იწვევს შეშფოთებათა დინამიკაში ახალი ტიპის ტრანზიენტულ არამდგრადობას მცირე $\mathrm{Ri} < 1$ (როდესაც სრულდება ე.წ. სიმეტრიული არამდგრადობის პირობა). კერძოდ, წრფივი თეორიის ფარგლებში, წანაცვლება და ბრუნვა იწვევს გრიგალური მოდის დროში ევოლუციას. სუფთა გრიგალური (აპერიოდული) შეშფოთებებს შეუძლიათ ამოქაჩონ ძირითადი დინების ენერგია, გაძლიერდნენ ტრანზიენტულად და შემდგომ მოახდინონ ტალღების გენერაცია.

LINEAR TRANSIENT DYNAMICS OF PERTURBATIONS IN NONGEOSTROPHIC FLOWS WITH A CONSTANT VERTICAL SHEAR./ Lominadze J., Chagelishvili G., Avsarkisov V./.Transactions of the Georgian Institute of Hydrometeorology. -2008. - т.115. – p. 330-343.- Eng.; Summ. Georg.; Eng.; Russ.

The linear dynamics of perturbations in unbounded nongeostrophic zonal inviscid flows with a constant vertical shear of velocity, when a fluid is incompressible and density is stably stratified along the vertical and meridional directions is investigated. Specific features of this dynamics are closely related to the non-normality of the linear operators governing perturbation evolution in shear flows. The roles of Coriolis parameter f (earth rotation) and shear rate A in the perturbation dynamics (instability) are analyzed. These two factors lead to a new transiently unstable type of perturbation dynamics for $\mathrm{Ri} < 1$ (i.e., when the condition for so-called symmetric instability is met). In particular, in the linear theory, shear and rotation causes the vortex mode to evolve with time. Pure vortex (aperiodic) perturbations are able to gain the basic flow energy, undergo transient amplification, and then generate waves.

Keywords: geophysical flow, geostrophic adjustment, potential vorticity, non-modal approach, algebraic instability

ЛИНЕЙНАЯ ТРАНЗИЕНТНАЯ ДИНАМИКА ВОЗМУЩЕНИЙ В НЕГЕОСТРОФИЧЕСКИХ ТЕЧЕНИЯХ С ПОСТОЯННЫМ ВЕРТИКАЛЬНЫМ СДВИГОМ./Ломинадзе Дж.Г., Чагелишвили Г.Д., Авсаркисов В.С./.Сб.Трудов Института Гидрометеорологии Грузии. –2008. – т.115. – с. 330-343. – Рус.; Рез. Груз., Анг.,Рус.

Исследуется линейная динамика возмущений в невязком неограниченном геострофическом зональном течении с постоянным вертикальным сдвигом скорости, при условии что жидкость несжимаема и плотность устойчиво стратифицированна по вертикальному и меридиональному направлениям. Особые свойства этой динамики тесно связанны с неортогональностью линейных операторов отвечающих за эволюцию возмущений в сдвиговых течениях. Исследованны роли параметра Кориолиса f (вращение земли) и величины сдвига A в динамике возмущения (неустойчивости). Эти два фактора ведут к новому транзиентно нестабильному типу динамики возмущений при $\mathrm{Ri} < 1$ (когда выполняются условия так называемой симметричной неустойчивости). В частности, в линейной теории, сдвиг и вращение приводят к эволюции вихревой моды со временем. Чисто вихревые (апериодические) возмущения способны приобретать энергию основного течения, подвергнуться транзиентному усилению, и затем сгенерировать волны